# Unit V: Game Theory - Games with Mixed Strategies

In certain cases, no pure strategy solutions exist for the game. In other words, saddle point does not exist. In all such game, both players may adopt an optimal blend of the strategies called **Mixed Strategy** to find a saddle point. The optimal mix for each player may be determined by assigning each strategy a probability of it being chosen. Thus these mixed strategies are probabilistic combinations of available better strategies and these games hence called **Probabilistic games**.

Games with mixed strategies can be solved either graphically or by linear programming. The graphical solution is suitable for games in which at least one player has exactly two pure strategies. The method is interesting because it explains the idea of a saddle point graphically. Linear programming can be used to solve any two-person zero-sum game.

**Graphical Solution of Games**: We start with the case of (2 X *n)* games in which player *A* has two strategies.

| | | $y_1$ | $y_2$ | ---- | $Y_j$ |
|---|---|---|---|---|---|
| | | B1 | B2 | ---- | $B_m$ |
| $x_1$ | A1 | $a_{11}$ | $a_{12}$ | ----- | $a_{1n}$ |
| $1- x_1$ | A2 | $a_{21}$ | $a_{22}$ | ----- | $a_{2n}$ |

The game assumes that player *A* mixes strategies $A_1$ and $A_2$ with the respective probabilities $x_1$ and $1 - x_1$ , $0 \leq x_1 \leq 1$ Player *B* mixes strategies $B_1$ to *Bn* with the probabilities $y_{1,}$ $y_2$, ----$y_n$ where $y_j \geq 0$ for j = 1,2,3,---,n and $y_{1+}$ $y_2$+ ----+$y_n$ = 1

In this case, *A's* expected payoff corresponding to *B's* jth pure strategy is computed as

$$(a_{1j} - a_{2j})\, x_1 - a_j,\ j = 1,2, \ldots , n$$

Player *A* thus seeks to determine the value of Xl that maximizes the minimum expected payoffs-that is,

$$\max_{x_1} \min_{j} \{(a_{1j} - a_{2j})x_1 - a_{2j}\}$$

The probabilistic mixed strategy games without saddle points are commonly solved by any of the following methods

| Sl. No. | Method | Applicable to |
|---|---|---|
| 1 | Analytical Method | 2x2 games |
| 2 | Dominance Property | Both for Mixed and Pure Strategy |
| 3 | Graphical Method | 2x2, mx2 and 2xn games |
| 4 | Simplex Method | 2x2, mx2, 2xn and mxn games |

## Analytical Method

A 2 x 2 payoff matrix where there is no saddle point can be solved by analytical method.

Given the matrix

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Value of the game is

$$V = \frac{a_{11}\, a_{22} - a_{21}\, a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

With the coordinates

$$x_1 = \frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} \quad , \quad x_2 = \frac{a_{11} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

$$y_1 = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} \quad , \quad y_2 = \frac{a_{11} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

**Alternative procedure to solve the strategy**

- Find the difference of two numbers in column 1 and enter the resultant under column 2. Neglect the negative sign if it occurs.
- Find the difference of two numbers in column 2 and enter the resultant under column 1. Neglect the negative sign if it occurs.
- Repeat the same procedure for the two rows.

**1. Solve**

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix} \end{array}$$

**Solution**

It is a 2 x 2 matrix and no saddle point exists. We can solve by analytical method

$$\begin{array}{ccc} & B & \\ A & \begin{bmatrix}5 & 1\\3 & 4\end{bmatrix} & \begin{matrix}1\\4\end{matrix} \\ & \begin{matrix}3 & \quad 2\end{matrix} & \end{array}$$

$$V = \frac{a_{11}\,a_{22} - a_{21}\,a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{20 - 3}{9 - 4}$$

V = 17 / 5

$S_A = (x_1, x_2) = (1/5, 4/5)$

$S_B = (y_1, y_2) = (3/5, 2/5)$

**2. Solve the given matrix**

$$\begin{array}{cc} & B \\ A & \begin{bmatrix}2 & -1\\-1 & 0\end{bmatrix} \end{array}$$

**Solution**

$$\begin{array}{ccc} & B & \\ A & \begin{bmatrix}2 & -1\\-1 & 0\end{bmatrix} & \begin{matrix}1\\3\end{matrix} \\ & \begin{matrix}1 & \quad 3\end{matrix} & \end{array}$$

$$V = \frac{a_{11}\,a_{22} - a_{21}\,a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{0 - 1}{2 + 2}$$

V = - 1 / 4

$S_A = (x_1, x_2) = (1/4, 3/4)$

$S_B = (y_1, y_2) = (1/4, 3/4)$